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$F = f(D, V, \rho, N, \mu)$ ۶-۴

$D \doteq L, \ V \doteq LT^{-1}, \ \rho \doteq FL^{-4}T^{2}, \ N \doteq T^{-1}, \ \mu \doteq FL^{-2}T, \ F \doteq F$

$F = (L)^{a} \times (LT^{-1})^{b} \times (FL^{-4}T^{2})^{c} \times (T^{-1})^{d} \times (FL^{-2}T)^{e}$

$$\left.\begin{aligned} 0 &= a + b - 4c - 2e \\ 0 &= -b + 2c - d + e \\ 1 &= c + e \end{aligned}\right\} \Rightarrow \begin{aligned} a &= 2 + d - e \\ b &= 2 - d - e \\ c &= 1 - e \end{aligned}$$

$$\Rightarrow F = f\left(D^{(2+d-e)} V^{(2-d-e)} \rho^{(1-e)} N^{d} \mu^{e}\right) = \rho V^{2} D^{2} f\left(\left(\frac{\rho V d}{\mu}\right)^{-e} \left(\frac{DN}{V}\right)^{d}\right)$$

$$\Rightarrow F = \rho V^{2} D^{2} f\left(\frac{\rho V d}{\mu}, \frac{DN}{V}\right)$$

$n = 4 \quad j = 3 \qquad F_D \doteq F, \ V \doteq LT^{-1}, \ \mu \doteq FL^{-2}T, \ D \doteq L$ ۶-۱۰. الف)

$n - j = 1 \Rightarrow \Pi_1 = F_D D^{a} V^{b} \mu^{c} \Rightarrow 0 = F \times (L)^{a} \times (LT^{-1})^{b} (FL^{-2}T)^{c}$

$\Rightarrow a, b, c = -1 \Rightarrow \Pi_1 = F_D D^{-1} V^{-1} \mu^{-1} \Rightarrow \dfrac{F_D}{D V \mu} = C$

$n = 4, \ j = 2 \qquad h \doteq D \doteq L, \ \sigma \doteq FL^{-1}, \ \gamma \doteq FL^{-3}$ ب)

$n - j = 2 \Rightarrow \Pi_1 = h D^{a} \gamma^{b}, \quad \Pi_2 = \sigma D^{c} \gamma^{d}$

$\pi_{۱} = L \times L^{a} \times (FL^{-۳})^{b} \Rightarrow \pi_{۱} = hD^{-1} \Rightarrow \frac{h}{D} = C$

$\pi_{۲} = FL^{-1} \times L^{c} \times L^{d} \Rightarrow \pi_{۲} = \sigma D^{-۲}\gamma^{-1} \Rightarrow \frac{\sigma}{D^{۲}\gamma} = C$

$\pi_{۱} = f(\pi_{۲}) \Rightarrow \frac{h}{D} = f\left(\frac{\sigma}{D^{۲}\gamma}\right)$

$n = ۶ \;,\; j = ۳ \quad \Delta h \doteq D \doteq h \doteq L, \; t \doteq T, \; \rho \doteq FL^{-۴}T^{۲}, \; \gamma \doteq FL^{-۳}$ (?

$\pi_{۱} = \Delta h D^{a} t^{b} \rho^{c} \;,\; \pi_{۲} = \gamma D^{e} t^{f} \rho^{g} \;,\; \pi_{۳} = h D^{h} t^{i} \rho^{j}$

$\pi_{۱} = L \times L^{a} T^{b} (FL^{-۴}T^{۲})^{c} \Rightarrow \pi_{۱} = \frac{\Delta h}{D}$

$\pi_{۲} = FL^{-۳} \times L^{e} \times T^{f} \times (FL^{-۴}T^{۲})^{g} \Rightarrow \pi_{۲} = \frac{\gamma t^{۲}}{\rho D}$

$\pi_{۳} = L \times L^{h} \times T^{i} \times (FL^{-۴}T^{۲})^{j} \Rightarrow \pi_{۳} = \frac{h}{D}$

$\pi_{۱} = f(\pi_{۲}, \pi_{۳}) \Rightarrow \frac{\Delta h}{D} = f\left(\frac{\gamma t^{۲}}{\rho D}, \frac{h}{D}\right)$

$\Delta P = f(\rho, \mu, D, L, \varepsilon, v)$ ۶_۱۴

$n = ۷ \;,\; j = ۳ \;,\; \Delta P \doteq F/L^{-۳}, \; \rho \doteq FL^{-۴}T^{۲}, \; \mu \doteq FL^{-۲}T, \; D \doteq L \doteq \varepsilon \doteq L$

$v \doteq LT^{-1}$

$n - j = ۴ \Rightarrow \pi_{۱} = v^{a_۱} D^{b_۱} \rho^{c_۱} \Delta P \Rightarrow \pi_{۱} = \frac{\Delta P}{\rho v^{۲} e^{۲}}$

$\pi_{۲} = v^{a_۱} D^{b_۲} \rho^{c_۲} \mu \Rightarrow \pi_{۲} = \frac{\mu}{\rho v \ell}$

$$\pi_3 = V^{a_3} D^{b_3} \rho^{c_3} \ell \Rightarrow \pi_3 = \frac{\ell}{D}$$

$$\pi_4 = V^{a_4} D^{b_4} \rho^{c_4} \varepsilon \Rightarrow \pi_4 = \frac{\varepsilon}{D}$$

$$\Rightarrow \frac{\Delta P}{\rho V^2} = f\left(Re, Fr, \frac{\varepsilon}{D}\right)$$

در سایر نسبت‌ها C ها، C ها باید متغیر بعددار دارد و فقط نسبت $\frac{\Delta P}{\rho V^2 \ell^2}$ بی‌بعد است.

$Q = f\left(D, \frac{\Delta H}{\ell}, \rho, \mu\right)$, $Q \doteq L^3T^{-1}$, $D \doteq L$, $\frac{\Delta H}{\ell} : 1$, $\rho \doteq ML^{-3}$ ۶-۲۱
$\mu \doteq ML^{-1}T^{-1}$

$n = 5, j = 3 \Rightarrow n - j = 2$

$$\Pi_1 = \rho^{a_1} \mu^{b_1} D^{c_1} Q \Rightarrow 0 = (ML^{-3})^{a_1} (ML^{-1}T^{-1})^{b_1} (L)^{c_1} (L^3T^{-1})$$

$$\Pi_2 = \rho^{a_2} \mu^{b_2} D^{c_2} \frac{\Delta H}{\ell} \Rightarrow 0 = (ML^{-3})^{a_2} (ML^{-1}T^{-1})^{b_2} (L)^{c_2} \times 1$$

$$\Rightarrow a_2 = b_2 = c_2 = 0 \Rightarrow C = \frac{Q\rho}{\mu D} \Rightarrow \frac{\Delta H}{\ell} = f\left(\frac{Q\rho}{\mu D}\right)$$

$V = f(\rho, D, w, b, d)$, $V \doteq LT^{-1}$, $\rho \doteq FL^{-4}T^2$, $D \doteq b \doteq d = L$, $w \doteq F$ ۶-۲۹

$$\xRightarrow{F} V = f\left(\frac{\rho}{\rho}, D, \frac{w}{\rho}, b, d\right) = f\left(D, \frac{w}{\underbrace{\rho}_{L^4L^{-2}}}, b, d\right)$$

$$\xRightarrow{T} V\sqrt{\frac{\rho}{w}} = f\left(D, \frac{w/\rho}{w/\rho}, b, d\right) = f(D, b, d)$$

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$$\overset{L}{\Rightarrow} vD\sqrt{\frac{\rho}{w}} = f\left(\frac{D}{D}, \frac{b}{D}, \frac{d}{D}\right) \Rightarrow vD\sqrt{\frac{\rho}{w}} = f\left(\frac{b}{D}, \frac{d}{D}\right)$$

$F_D = f(D, v, \rho, \mu)$, $F_D \doteq F, D \doteq L, v \doteq LT^{-1}, \rho \doteq FL^{-4}T^{2}, \mu \doteq FL^{-2}T$ ۶_۳۴

$$\left.\begin{array}{l} D \doteq L \\ v \doteq LT^{-1} \doteq DT^{-1} \\ \rho \doteq FL^{-4}T^{2} \doteq FD^{-4}\left(\frac{D}{v}\right)^{2} = FD^{-2}v^{-2} \end{array}\right\} \begin{array}{l} L \doteq D \\ \Rightarrow T \doteq \frac{D}{v} \\ F \doteq \rho D^{2}v^{2} \end{array}$$

$$\left.\begin{array}{l} F_D \doteq F \doteq D^{2}v^{2}\rho \Rightarrow \pi_1 = \frac{F_D}{D^{2}v^{2}\rho} \\ \mu \doteq FL^{-2}T \doteq (\rho D^{2}v^{2})D^{-2}\left(\frac{D}{v}\right) = \rho Dv \rightarrow \pi_2 = \frac{\mu}{\rho Dv} \end{array}\right\} \Rightarrow \pi_1 = f(\pi_2)$$

$$\Rightarrow \frac{F_D}{D^{2}v^{2}\rho} = f\left(\frac{\mu}{\rho Dv}\right) \rightarrow F_D = \rho v^{2}D^{2} f\left(\frac{\mu}{\rho Dv}\right)$$

عدد فرود : $\left(\frac{v}{\sqrt{gl}}\right)_m = \left(\frac{v}{\sqrt{gl}}\right)_p \Rightarrow \frac{v_p}{v_m} = \sqrt{\frac{l_p}{l_m}} = (L_r)^{-\frac{1}{2}}$ ۶_۴۵

عدد وبر : $\left(\frac{\rho v^{2}l}{\sigma}\right)_m = \left(\frac{\rho v^{2}l}{\sigma}\right)_p \Rightarrow \frac{\sigma_p}{\sigma_m} = \left(\frac{v_p}{v_m}\right)^{2} L_r^{-1}$

$$\frac{\sigma_p}{\sigma_m} = \left(L_r^{-\frac{1}{2}}\right)^{2} L_r^{-1} = L_r^{-2} = \left(\frac{1}{10}\right)^{-2} = 100$$

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$\tau_w = f(D, Q, \mu, \rho)$, $\tau_w : FL^{-۲}$, $D : L$, $Q : L^{۳}T^{-۱}$, $\rho : FL^{-۴}T^{۲}$ ۶_۵۱
$\mu : FL^{-۲}T$

روش گام به گام :

$\xrightarrow{F} \frac{\tau_w}{\rho} = f_۱\left(D, Q, \frac{\mu}{\rho}\right)$

$\xrightarrow{L} \frac{\tau_w}{\rho D^۲} = f_۲\left(\frac{Q}{D^۳}, \frac{\mu}{\rho D^۲}\right)$

$\xrightarrow{T} \frac{\tau_w}{\rho D^۲}\left(\frac{D^۳}{Q}\right)^۲ = f_۳\left(\frac{Q}{D^۳}\left(\frac{D^۳}{Q}\right), \frac{\mu}{\rho D^۲}\left(\frac{D^۳}{Q}\right)\right)$

$\Rightarrow \frac{\tau_w \cdot D^۴}{\rho Q^۲} = f\left(\frac{\nu D}{Q}\right)$

$\left(\frac{\nu D}{Q}\right)_m = \left(\frac{\nu D}{Q}\right)_p \Rightarrow \frac{Q_m}{Q_p} = \frac{D_m}{D_p} \times \cancel{\frac{\nu_m}{\nu_p}}^{۱} = \frac{D_m}{D_p}$

$\Rightarrow Q_m = Q_p D_m / D_p = \frac{۱/۲ \times ۰/۲}{۰/۳} = ۰/۸ \ \frac{m^۳}{s} \xrightarrow[\text{نمودار}]{\text{طبق}} \tau_{w_m} = ۰/۱۷ Pa$

$\left(\frac{\tau_w D^۴}{\rho Q^۲}\right)_m = \left(\frac{\tau_w D^۴}{\rho Q^۲}\right)_p \Rightarrow \tau_{w_p} = \tau_{w_m}\left(\frac{D_m}{D_p}\right)^۴\left(\frac{Q_p}{Q_m}\right)^۲\left(\frac{\rho_p}{\rho_m}\right)$

$\Rightarrow \tau_{w_p} = \tau_{w_m}\left(\frac{D_m}{D_p}\right)^۴\left(\frac{D_p}{D_m}\right)^۲\underbrace{\cancel{\left(\frac{\rho_p}{\rho_m}\right)}}_{۱} = ۰/۱۷ \times \left(\frac{۰/۲}{۰/۳}\right)^۲ = ۰/۰۷۵۶ \ Pa$

۶_۵۴

$۲۰ ^\circ C \Rightarrow \nu = ۱/۰۰۷ \times ۱۰^{-۶} \ m^۲/s$

عدد رینولدز : $\frac{V_m}{V_p} = \left(\frac{\nu_m}{\nu_p}\right)\left(\frac{D_p}{D_m}\right) \Rightarrow \frac{V_m}{۲۵} = \left(\frac{۱/۰۰۷ \times ۱۰^{۶}}{۰/۰۰۰۲ \times ۴۰}\right) \times \left(\frac{۱/۲}{D_m}\right)$

$$\Rightarrow V_m = \frac{۴/۰۴۲}{D_m}$$

$$Q_m = V_m A_m \Rightarrow ۰/۰۷۵ = \frac{۴/۰۴۲}{D_m} \times \frac{\pi}{۴} D_m^{۲} \Rightarrow D_m = ۰/۱۴\ m$$

$$L_r = \frac{D_m}{D_p} = \frac{۰/۱۲}{۱/۲} = \frac{۱}{۷۴}$$

۶_۶۰

$$V_m = ۱۵\ m/s \xRightarrow{\text{عدد ر}} T_m = ۱۸۰\ N.m$$

$$\frac{T_p}{T_m} = \frac{F_{D_p} l_p}{F_{D_m} l_m} = ۱ \times \frac{l_p}{l_m} = (L_r)^{-۱} \Rightarrow T_p = T_m \frac{۱}{L_r} = ۱۸۰ \times \frac{۱}{\frac{۱}{۲۰}} \Rightarrow T_p = ۳۶۰۰\ N.m$$

۶_۶۵ - تشبیه سازی براساس برابری هردو اعداد رینولدز و فرود : رابطه بین لزجت

سیالات مدل به جم واقعی براساس رابطه $\frac{\nu_m}{\nu_p} = \left(\frac{l_m}{l_p}\right)^{\frac{۳}{۲}} = L_r^{\frac{۳}{۲}}$ بدست می‌آید.

پس گزینه ۴ صحیح است

اثبات

$$\Rightarrow Re_p = \frac{V_p l_p}{\nu_p} = Re_m = \frac{V_m l_m}{\nu_m} \Rightarrow \frac{V_m}{V_p} = \frac{\nu_m}{\nu_p} \times \frac{l_p}{l_m} \quad ①$$

$$Fr_p = \frac{V_p}{\sqrt{g l_p}} = Fr_m = \frac{V_m}{\sqrt{g l_m}} \Rightarrow \frac{V_m}{V_p} = \sqrt{\frac{l_m}{l_p}} \quad ②$$

$$\xRightarrow{۱ و ۲} \frac{\nu_m}{\nu_p} = \left(\frac{l_m}{l_p}\right)^{\frac{۱}{۲}} \times \frac{l_m}{l_p} = \left(\frac{l_m}{l_p}\right)^{\frac{۳}{۲}}$$

۶_۷۱

مجاری روباز ⇐ عدد فرود ⇐ $\frac{t_m}{t_p} = \sqrt{L_r}$

$$\Rightarrow \frac{\omega_p}{\omega_m} = \frac{1/t_p}{1/t_m} = \frac{t_m}{t_p} = \sqrt{L_r} \Rightarrow \omega_p = \omega_m \sqrt{L_r} = n\sqrt{\frac{1}{s}} = \frac{n}{\sqrt{s}} \Rightarrow$$ گزینه ۴