6-3) $F_D = \phi(D, V, \rho, \mu)$

$F_D = MLT^{-2}$ $D = L$ $V = LT^{-1}$ $\rho = ML^{-3}$ $\mu = ML^{-1}T^{-1}$

$$MLT^{-2} = L^a \times (LT^{-1})^b \times (ML^{-3})^c \times (ML^{-1}T^{-1})^d$$

$L \to 1 = a + b - 3c - d$ $M \to 1 = c + d$ $T \to -2 = -b - d \to 2 = b + d$

سه معادله و ۴ مجهول ← توان‌ها را برحسب d می‌نویسیم

$a = 2 - d$

$b = 2 - d$

$c = 1 - d$ $F_D = D^{2-d} \times V^{2-d} \times \rho^{1-d} \times \mu^{d} = D^2 V^2 \rho \left(\frac{\mu}{DV\rho}\right)^d$

به دلیل اینکه $\frac{\mu}{DV\rho}$ عددی بی‌بعد است توان آن را می‌توان در نظر گرفت

مثال ۹-۴ :

$$y_1 = f(y_1, V_1, g, \rho, \nu, \varepsilon)$$

$$y_2 = f(y_1^a V_1^b g^c \rho^d \nu^e \varepsilon^f)$$

$$y_2 \doteq y_1 \doteq \varepsilon \doteq L \qquad V \doteq LT^{-1} \qquad g \doteq LT^{-2}$$

$$\rho \doteq FL^{-4}T^{2} \qquad \nu \doteq L^{2}T^{-1}$$

$$L \doteq (L)^a (LT^{-1})^b (LT^{-2})^c (FL^{-4}T^{2})^d (L^{2}T^{-1})^e (L)^f$$

$$\begin{cases} F: & 0 = d \\ L: & 1 = a+b+c-4d+2e+f \\ T: & 0 = -b-2c-e \end{cases} \Rightarrow \begin{cases} d = 0 \\ e = -b-2c \\ f = 1-a+b+3c \end{cases}$$

$$\Rightarrow y_2 = f\left[y_1^{(a)}\, \upsilon_1^{(b)}\, g^{(c)}\, \rho^{(0)}\, \nu^{(-b-2c)}\, \varepsilon^{(1-a+b+3c)}\right]$$

$$\underbrace{\frac{y_2}{\varepsilon}}_{\Pi_1} = f\left[\underbrace{\left(\frac{y_1}{\varepsilon}\right)}_{\Pi_2}^{a} \underbrace{\left(\frac{\upsilon_1 \varepsilon}{\nu}\right)}_{\Pi_3}^{b} \underbrace{\left(\frac{g\varepsilon^3}{\nu^2}\right)}_{\Pi_4}^{c}\right]$$

$$\Pi_1' = \Pi_1 \frac{1}{\Pi_2} = \left(\frac{y_2}{\varepsilon}\right)\left(\frac{\varepsilon}{y_1}\right) = \frac{y_2}{y_1} \quad \Rightarrow \Pi_2' = \frac{1}{\Pi_2} = \frac{\varepsilon}{y_1}$$

$$\Pi_3' = \sqrt{\Pi_3^{2} \frac{1}{\Pi_4} \frac{1}{\Pi_2}} = \sqrt{\left(\frac{\upsilon_1^2 \varepsilon^2}{\nu^2}\right)\left(\frac{\nu^2}{g\varepsilon^3}\right)\left(\frac{\varepsilon}{y_1}\right)} = \frac{\upsilon_1}{\sqrt{g y_1}} = Fr_1$$

$$\Rightarrow \Pi_4' = \Pi_2 \Pi_3 = \left(\frac{y_1}{\varepsilon}\right)\left(\frac{\upsilon_1 \varepsilon}{\nu}\right) = \frac{\upsilon_1 y_1}{\nu} = Re_1$$

$$\frac{y_2}{y_1} = f\left[\left(\frac{\varepsilon}{y_1}\right)^a (Fr_1)^b (Re_1)^c\right] \Rightarrow$$

$$\frac{y_2}{y_1} = f\left(Fr_1, Re_1, \frac{\varepsilon}{y_1}\right)$$

۱۳-۶ :

با استفاده از روش باکینگهام :

$$\Rightarrow h = f(\omega, \rho, g, R) \Rightarrow n = 5$$

ابعاد متغیرها $\Rightarrow h \doteq R \doteq L \quad g \doteq LT^{-1} \quad \rho \doteq FL^{-4}T^{2} \quad \omega \doteq T^{-1}$

$$T, F, L \Rightarrow j = 3 \qquad \Rightarrow k = n - j = 5 - 3 = 2$$

با توجه به شروط روش باکینگهام در انتخاب متغیرهای تکراری : ρ, ω, R

$$\left.\begin{aligned} \Pi_1 &= h R^{a_1} \omega^{b_1} \rho^{c_1} \\ \Pi_2 &= g R^{a_2} \omega^{b_2} \rho^{c_2} \end{aligned}\right\} \Rightarrow \begin{aligned} \Pi_1 &\doteq \left[L (L)^{a_1} (T^{-1})^{b_1} (ML^{-3})^{c_1}\right] \\ \Pi_2 &\doteq \left[(LT^{-2})(L)^{a_2}(T^{-1})^{b_2}(ML^{-3})^{c_2}\right] \end{aligned}$$

$$\Rightarrow \begin{cases} \Pi_1 = \dfrac{h}{R} \\ \Pi_2 = \dfrac{g}{R\omega^2} \end{cases} \qquad \frac{h}{R} = f\left(\frac{g}{R\omega^2}\right)$$

6-19) $f_K = f(v, \rho, \mu, D, c)$ $n = 6$ $j = 3$ $\rightarrow K = 3$

$f_K = T^{-1}$, $v = LT^{-1}$, $\rho = ML^{-3}$, $\mu = ML^{-1}T^{-1}$, $D = L$, $C = LT^{-1}$

$\Pi_1 = f_K \rho^{a_1} v^{b_1} D^{c_1}$ $\quad T^{-1} \times (ML^{-3})^{a_1} \times (LT^{-1})^{b_1} \times (L)^{c_1}$

$\Pi_2 = \mu \rho^{a_2} v^{b_2} D^{c_2}$ $\quad (ML^{-1}T^{-1}) \times (ML^{-3})^{a_2} \times (LT^{-1})^{b_2} \times (L)^{c_2}$

$\Pi_3 = c \rho^{a_3} v^{b_3} D^{c_3}$ $\quad (LT^{-1}) \times (ML^{-3})^{a_3} \times (LT^{-1})^{b_3} \times (L)^{c_3}$

$\Pi_1 \rightarrow -1 - b_1 = 0$, $a_1 = 0$, $-3a_1 + b_1 + c_1 = 0 \rightarrow b_1 = -1, a_1 = 0, c_1 = +1$

$\Pi_2 \rightarrow 1 + a_2 = 0$, $-1 - 3a_2 + b_2 + c_2 = 0$, $-1 - b_2 = 0 \rightarrow b_2 = -1, a_2 = -1, c_2 = -1$

$\Pi_3 \rightarrow 1 - 3a_3 + b_3 + c_3 = 0$, $a_3 = 0$, $-1 - b_3 = 0 \rightarrow b_3 = -1, a_3 = 0$ $c_3 = 0$

$$\begin{cases} a_1 = 0 \\ b_1 = -1 \\ c_1 = +1 \end{cases} \qquad \begin{cases} a_2 = -1 \\ b_2 = -1 \\ c_2 = -1 \end{cases} \qquad \begin{cases} a_3 = 0 \\ b_3 = -1 \\ c_3 = 0 \end{cases}$$

$\Pi_1 = f_K \rho^0 v^{-1} D^{+1} = \dfrac{f_K D}{v} \rightarrow \dfrac{T^{-1} \times L}{LT^{-1}} = 0$ ✓

$\Pi_2 = \mu \rho^{-1} v^{-1} D^{-1} = \dfrac{\mu}{\rho v D} \rightarrow$ عدد رینولدز ✓

$\Pi_3 = c \rho^0 v^{-1} D^0 = \dfrac{c}{v} \rightarrow \dfrac{LT^{-1}}{LT^{-1}} = 0$ ✓

$n = f(\tau_0, \rho, g\Delta\rho, \mu, d)$ $n \doteq L^{2}T^{-1}$ $\tau_0 = FL^{-2}$ $\rho \doteq FL^{-4}T^{2}$ ۶-۳۴)

$g\Delta\rho \doteq (LT^{-2})(FL^{-4}T^{2}) \doteq FL^{-3}$ $\mu \doteq FL^{-2}T$ $d \doteq L$ $\Rightarrow$ d, ρ, τ_0

متغیرهای تکراری

$n = 4$ $j = 3$ $k = 3 \rightarrow$ تا پی

$\Pi_1 = n\, d^{a_1} \tau_0^{b_1} \rho^{c_1} \Rightarrow (L^{2}T^{-1})(L^{a_1})(FL^{-2})^{b_1}(FL^{-4}T^{2})^{c_1} \doteq \Pi_1 \Rightarrow \dfrac{n\,d^{2}}{\sqrt{\tau_0/\rho}}$

$\Pi_2 = (g\Delta\rho)\, d^{a_2} \tau_0^{b_2} \rho^{c_2} \Rightarrow \Pi_2 \doteq (FL^{-3})(L^{a_2})(FL^{-2})^{b_2}(FL^{-4}T^{2})^{c_2} \Rightarrow \dfrac{d\, g\Delta\rho}{\tau_0}$

$\Pi_3 = \mu\, d^{a_3} \tau_0^{b_3} \rho^{c_3} \Rightarrow \Pi_3 \doteq (FL^{-2}T)(L)^{a_3}(FL^{-2})^{b_3}(FL^{-4}T^{2})^{c_3} \Rightarrow \dfrac{v}{d\sqrt{\tau_0/\rho}}$

$\Rightarrow n d^{2}\sqrt{\dfrac{\rho}{\tau_0}} = f\left(\dfrac{d g\Delta\rho}{\tau_0}, \dfrac{v}{d}\sqrt{\dfrac{\rho}{\tau_0}}\right) = f\left(\dfrac{d\sqrt{\tau_0/\rho}}{v}, \dfrac{\tau_0}{g\Delta\rho\, d}\right)$

6-28)

$f_K = f(v, \rho, \mu, D)$

$f_K = T^{-1}$ $v = LT^{-1}$ $\rho = ML^{-3}$ $\mu = ML^{-1}T^{-1}$ $D = L$

$\rho \rightarrow f_K = f\left(v, \dfrac{\rho}{\rho}, \underbrace{\dfrac{\mu}{\rho}}_{L^{2}T^{-1}}, D\right)$ $v \rightarrow \underbrace{\dfrac{f_K}{v}}_{L^{-1}} = f\left(v/v, \underbrace{\dfrac{\mu}{\rho v}}_{L}, D\right)$

$D \rightarrow \dfrac{f_K D}{v} = f\left(\dfrac{\mu}{\rho v D}, \dfrac{D}{D}\right) \rightarrow \dfrac{f_K D}{v} = f\left(\dfrac{\mu}{\rho v D}\right)$

6-34) $F_D = f(D, V, \rho, \mu)$

$$\begin{cases} D \doteq L \\ V \doteq LT^{-1} \doteq DT^{-1} \\ \rho \doteq ML^{-3} \doteq MD^{-3} \end{cases} \qquad \begin{cases} L \doteq D \\ T \doteq DV^{-1} \\ M \doteq \rho D^3 \end{cases}$$

$$F_D = MLT^{-2} \doteq \rho D^3 \times D \times (DV^{-1})^{-2} = \rho D^2 V^2 \rightarrow \frac{F_D}{\rho D^2 V^2}$$

$$\mu = ML^{-1}T^{-1} \doteq \rho D^3 \times D^{-1} \times D^{-1} V^1 = \rho D V \rightarrow \frac{\mu}{\rho V D}$$

6-39) $\Delta p = f(\sigma, R)$

$\Delta p = FL^{-2}$ - $\sigma = FL^{-1}$ - $R = L$

$(FL^{-2}) = (FL^{-1})^a \times L^b \rightarrow F \rightarrow 1 = a$, $L \rightarrow -2 = -a + b \rightarrow b = -1$

$\Delta p = \sigma^1 \times R^{-1} \rightarrow \frac{\Delta p R}{\sigma} = \Pi$

Δp	5.,8	63,2	72,3	126,9
R	0,.64	0,.51	0,.44	0,.25
Π	44,53	44,15	43,58	43,57

$\bar{\Pi} = 43,9$

$$\frac{\Delta p R}{\sigma} = 43,9 \rightarrow \Delta p = 43,9 \frac{\sigma}{R}$$

6-47)

$$v = f(\Delta p, d, l, \mu) \qquad n=5 \quad j=3 \quad k=2$$

$$v = LT^{-1} \quad \Delta p = ML^{-1}T^{-2} \quad d = l = L \quad \mu = ML^{-1}T^{-1}$$

$$\Pi_1 = v\,\Delta p^{a_1} d^{b_1} \mu^{c_1} \rightarrow LT^{-1} \times (ML^{-1}T^{-2})^{a_1} \times (L)^{b_1} \times (ML^{-1}T^{-1})^{c_1}$$

$$\Pi_2 = l\,\Delta p^{a_2} d^{b_2} \mu^{c_2} \rightarrow L \times (ML^{-1}T^{-2})^{a_2} \times (L)^{b_2} \times (ML^{-1}T^{-1})^{c_2}$$

$$\rightarrow \Pi_1 \rightarrow L \rightarrow 1 - a_1 + b_1 - c_1 = 0 \quad b_1 = -1$$

$$T \rightarrow -1 - 2a_1 - c_1 = 0 \rightarrow a_1 = -1$$

$$M \Rightarrow a_1 + c_1 = 0 \rightarrow a_1 = -c_1 \rightarrow c_1 = 1$$

$$\Pi_2 \rightarrow 1 - a_2 + b_2 - c_2 = 0 \quad b_2 = -1$$

$$a_2 + c_2 = 0 \rightarrow a_2 = -c_2 \rightarrow a_2 = c_2 = 0$$

$$-2a_2 - c_2 = 0 \rightarrow -2a_2 = c_2$$

$$\Pi_1 = v\,\Delta p^{-1} d^{-1} \mu^{+1} \rightarrow \frac{v\mu}{\Delta p\, d}$$

$$\Pi_2 = l \times \Delta p^{0} \times d^{-1} \times \mu^{0} \rightarrow l/d$$

$$\left(\frac{v\mu}{d\Delta p}\right)_m = \left(\frac{v\mu}{d\Delta p}\right)_p$$

خاصیت سیال را در نمونه و اصل یکسان در نظر میگیریم

در نتیجه $\mu_m = \mu_p$

$$\frac{v_m}{v_p} = \frac{d_m}{d_p} \times \frac{\Delta p_m}{\Delta p_p} = 1/4 \times 1/2 = 1/8$$

مقیاس سرعت

(6-54) $Re = \frac{\mu}{\rho V D}$

فرض می‌کنیم مشخصات روغن در نمونه و اصل یکسان باشد

یعنی $\rho_m = \rho_p$ و $\mu_m = \mu_p$

$Re_m = Re_p \rightarrow \frac{V_m}{V_p} = \frac{D_p}{D_m}$

$\frac{1{,}5\ cm/s}{V_p} = 5 \rightarrow V_p = 0{,}3 \rightarrow$ گزینه 1

(6-60) از روی نمودار $T_m = 180$ مشخصات سیال را در نمونه و اصل یکسان در نظر می‌گیریم

$\frac{T_p}{T_m} = \frac{F_{Dp}\, l_p}{F_{Dm}\, l_m} = 1 \times \frac{l_p}{l_m} = \frac{1}{l_r} \longrightarrow T_p = T_m \times 1/l_r \rightarrow T_p = 3600\ N\cdot m$

(6-68) $\frac{Q_m}{Q_p} = 25^{5/2}$ $Q_m = 0{,}6\ m^3/s \rightarrow Q_p = \frac{Q_m}{25^{5/2}} \rightarrow Q_p = 1{,}92 \times 10^{-4}\ m^3/s$

گزینه 2