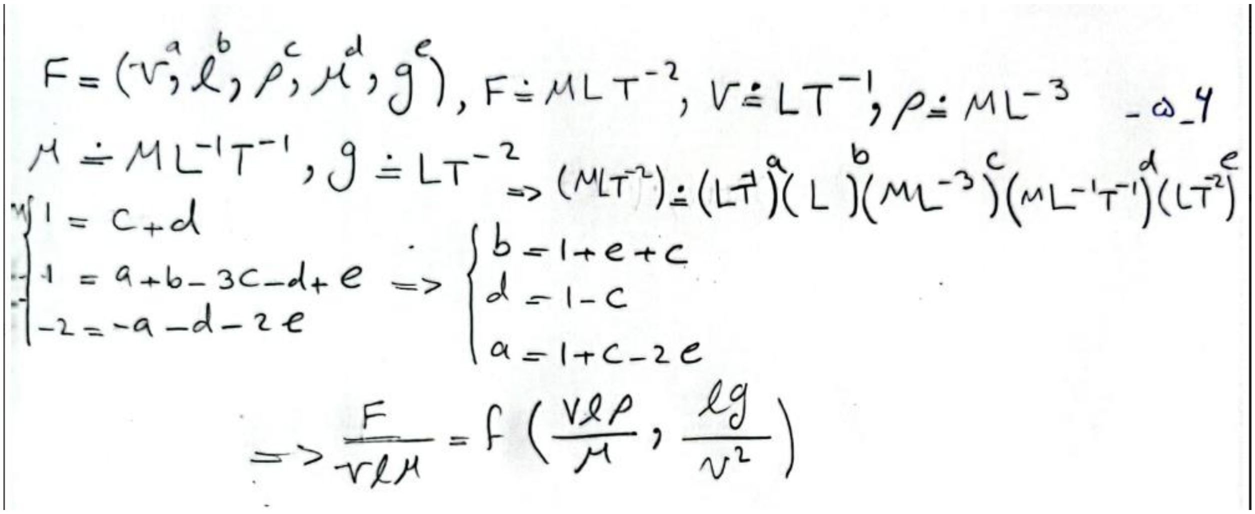


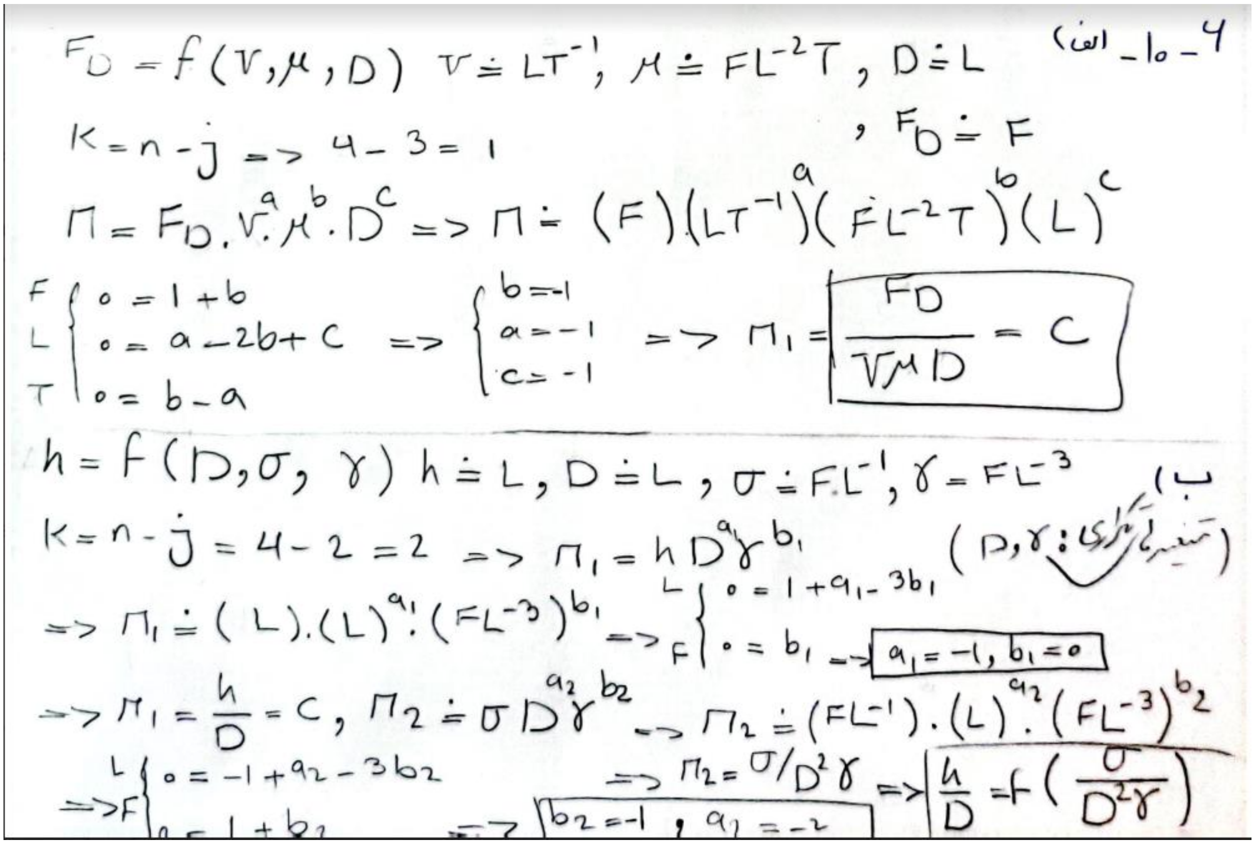
۴-۵-

$$F = (V^a, l^b, \rho^c, \mu^d, g^e),\ F \doteq MLT^{-2},\ V \doteq LT^{-1},\ \rho \doteq ML^{-3}$$

$$\mu \doteq ML^{-1}T^{-1},\ g \doteq LT^{-2} \Rightarrow (MLT^{-2}) \doteq (LT^{-1})^a (L)^b (ML^{-3})^c (ML^{-1}T^{-1})^d (LT^{-2})^e$$

$$\begin{array}{l} M \\ L \\ T \end{array}\begin{cases} 1 = c + d \\ 1 = a + b - 3c - d + e \\ -2 = -a - d - 2e \end{cases} \Rightarrow \begin{cases} b = 1 + e + c \\ d = 1 - c \\ a = 1 + c - 2e \end{cases}$$

$$\Rightarrow \frac{F}{Vl\mu} = f\left(\frac{Vl\rho}{\mu}, \frac{lg}{V^2}\right)$$



۴-۱۰- الف)

$$F_D = f(V, \mu, D) \quad V \doteq LT^{-1},\ \mu \doteq FL^{-2}T,\ D \doteq L$$

$$K = n - j \Rightarrow 4 - 3 = 1 \qquad ,\ F_D \doteq F$$

$$\Pi = F_D \cdot V^a \cdot \mu^b \cdot D^c \Rightarrow \Pi \doteq (F)(LT^{-1})^a (FL^{-2}T)^b (L)^c$$

$$\begin{array}{l} F \\ L \\ T \end{array}\begin{cases} 0 = 1 + b \\ 0 = a - 2b + c \\ 0 = b - a \end{cases} \Rightarrow \begin{cases} b = -1 \\ a = -1 \\ c = -1 \end{cases} \Rightarrow \Pi_1 = \boxed{\frac{F_D}{V\mu D} = C}$$

ب)

$$h = f(D, \sigma, \gamma) \quad h \doteq L,\ D \doteq L,\ \sigma \doteq FL^{-1},\ \gamma = FL^{-3}$$

$$K = n - j = 4 - 2 = 2 \Rightarrow \Pi_1 = h D^{a_1} \gamma^{b_1}$$ (D, γ: متغیرهای تکراری)

$$\Rightarrow \Pi_1 \doteq (L).(L)^{a_1} \cdot (FL^{-3})^{b_1} \Rightarrow \begin{array}{l} L \\ F \end{array}\begin{cases} 0 = 1 + a_1 - 3b_1 \\ 0 = b_1 \end{cases} \Rightarrow \boxed{a_1 = -1,\ b_1 = 0}$$

$$\Rightarrow \Pi_1 = \frac{h}{D} = C,\ \Pi_2 \doteq \sigma D^{a_2} \gamma^{b_2} \Rightarrow \Pi_2 \doteq (FL^{-1}).(L)^{a_2} \cdot (FL^{-3})^{b_2}$$

$$\Rightarrow \begin{array}{l} L \\ F \end{array}\begin{cases} 0 = -1 + a_2 - 3b_2 \\ 0 = 1 + b_2 \end{cases} \Rightarrow \boxed{b_2 = -1,\ a_2 = -2} \Rightarrow \Pi_2 = \sigma / D^2 \gamma \Rightarrow \boxed{\frac{h}{D} = f\left(\frac{\sigma}{D^2 \gamma}\right)}$$

ج)

$\Delta h = f(t, \rho, D, \gamma, h)\ ,\ t \doteq T\ ,\ \Delta h \doteq D \doteq h \doteq L$

$\rho \doteq ML^{-3} \doteq FL^{-4}T^{2}\ ,\ \gamma = FL^{-3} \Rightarrow K = n - j = 6 - 3 = 3$

$\Rightarrow \Pi_1 = \Delta h\, D^{a_1} \cdot \rho^{b_1} \cdot t^{c_1} \Rightarrow \Pi_1 \doteq (L)(L)^{a_1}(FL^{-4}T^{2})^{b_1}(T)^{c_1}$ (متغیرهای تکراری ρ و D و t هستند)

$$\Rightarrow \begin{array}{c|l} F & 0 = b_1 \\ L & 0 = 1 + a_1 - 4b_1 \\ T & 0 = 2b_1 + c_1 \end{array} \Rightarrow b_1 = 0\ ,\ c_1 = 0\ ,\ a_1 = -1 \Rightarrow \boxed{\Pi_1 = \frac{\Delta h}{D}}$$

$\Rightarrow \Pi_2 = \gamma D^{a_2} \cdot \rho^{b_2} \cdot t^{c_2} \Rightarrow \Pi_2 = (FL^{-3}) \cdot (L)^{a_2} \cdot (FL^{-4}T^{2})^{b_2}(T)^{c_2}$

$$\Rightarrow \begin{array}{c|l} F & 0 = 1 + b_1 \\ L & 0 = -3 + a_2 - 4b_1 \\ T & 0 = 2b_2 + C_2 \end{array} \Rightarrow b_1 = -1\ ,\ C_2 = 2\ ,\ a_2 = -1 \Rightarrow \boxed{\Pi_2 = \frac{\gamma t^2}{D\rho}}$$

$\Rightarrow \Pi_3 = h D^{a_3} \rho^{b_3} t^{c_3} \Rightarrow \Pi_3 = (L) \cdot (L)^{a_3} \cdot (FL^{-4}T^{2})^{b_3}(T)^{c_3}$

$$\Rightarrow \begin{array}{c|l} F & 0 = b_3 \\ L & 0 = 1 + a_3 - 4b_3 \\ T & 0 = C_3 \end{array} \Rightarrow \left| \begin{array}{l} b_3 = C_3 = 0 \\ a_3 = -1 \end{array} \right. \Rightarrow \boxed{\Pi_3 = \frac{h}{D}}$$

$$\Rightarrow \Pi_1 = f(\Pi_2, \Pi_3) \Rightarrow \frac{\Delta h}{D} = \left(\frac{\gamma t^2}{D\rho}, \frac{h}{D} \right)$$

$f_k = f(V, \rho, \mu, D) \Rightarrow f_k \doteq T^{-1},\ V \doteq LT^{-1},\ \rho \doteq FL^{-4}T^{2}$ 15-6

$\mu \doteq FL^{-2}T,\ D \doteq L \Rightarrow k = n - j \Rightarrow 5 - 3 = 2$

$\Rightarrow \Pi_1 = f_k \cdot V^{a_1} \cdot \mu^{b_1} \cdot D^{c_1} \Rightarrow \Pi_1 \doteq (T^{-1}).(LT^{-1})^{a_1}.(FL^{-2}T)^{b_1}.(L)^{c_1}$ (متغیر های تکراری V ، μ ، D هستند)

$$\Rightarrow \begin{array}{c|l} F & 0 = b_1 \\ L & 0 = a_1 - 2b_1 + c_1 \\ T & 0 = -1 - a_1 + b_1 \end{array} \Rightarrow \left\{ \begin{array}{l} b_1 = 0 \\ a_1 = -1 \\ c_1 = 1 \end{array} \right. \Rightarrow \Pi_1 = \frac{f_k D}{V}$$

ادامه در صفحه بعد →

(v)

ادامه ۱۵-۶-

$$, \Pi_2 = \rho V^{a_2} \mu^{b_2} D^{c_2} \Rightarrow \Pi_2 \doteq (FL^{-4}T^{2}).(LT^{-1})^{a_2}.(FL^{-2}T)^{b_2}.(L)^{c_2}$$

$$\Rightarrow \begin{array}{c|l} F & 0 = 1 + b_2 \\ L & 0 = -4 + a_2 - 2b_2 + c_2 \\ T & 0 = 2 - a_2 + b_2 \end{array} \Rightarrow \left\{ \begin{array}{l} b_2 = -1 \\ a_2 = 1 \\ c_2 = 1 \end{array} \right. \Rightarrow \Pi_2 = \frac{\rho V D}{\mu}$$

→ (Re عدد رینولدز)

$$\Rightarrow \Pi_1 = f(\Pi_2) \Rightarrow \frac{f_k D}{V} = f\left(\frac{\rho V D}{\mu}\right)$$

$f\left(Q, D, \frac{\Delta H}{l}, \rho, \mu\right) \Rightarrow \Pi_1 = \frac{\Delta H}{l},\ Q \doteq L^{3}T^{-1}$ ۲۱-۴-

$D \doteq L,\ \rho \doteq FL^{-4}T^{2},\ \mu = FL^{-2}T \Rightarrow \Pi_2 = Q D^{a} \rho^{b} \mu^{c}$

$\Rightarrow \Pi_2 \doteq (L^{3}T^{-1}).(L)^{a}.(FL^{-4}T^{2})^{b}.(FL^{-2}T)^{c}$

$$\Rightarrow \begin{array}{c|l} F & 0 = b + c \\ L & 0 = 3 + a - 4b - 2c \\ T & 0 = -1 + 2b + c \end{array} \Rightarrow \left\{ \begin{array}{l} b = 1 \\ c = -1 \\ a = -1 \end{array} \right. \quad \Pi_2 = \frac{Q \rho}{\mu D}$$

۶-۲۸-

۱- انتخاب μ برای حذف F :

(متغیرهای تکراری v, μ, D)

$$F_K = f(v, \rho, \mu, D)$$

$$\underset{T^{-1}}{F_K}\quad \underset{LT^{-1}}{v}\quad \underset{FL^{-4}T^{2}}{\rho}\quad \underset{FL^{-2}T}{\mu}\quad \underset{L}{D}$$

۲- انتخاب T برای حذف T :

$$\underset{T^{-1}}{F_K} = f\left(\underset{LT^{-1}}{v}, \underset{L^{-2}T}{\frac{\rho}{\mu}}, \underset{\phi}{\frac{\mu}{\mu}}, \underset{L}{D}\right) = f\left(\underset{LT^{-1}}{v}, \underset{FL^{-2}T}{\frac{\rho}{\mu}}, \underset{L}{D}\right)$$

۳- انتخاب D برای حذف L :

$$\Rightarrow \underset{L^{-1}}{\frac{F_K}{v}} = f\left(\underset{\phi}{\frac{v}{v}}, \underset{L^{-1}}{\frac{\rho v}{\mu}}, \underset{L}{D}\right) = f\left(\underset{FL^{-1}}{\frac{\rho v}{\mu}}, \underset{L}{D}\right)$$

عدد رینولدز

$$\Rightarrow \underset{\phi}{\frac{F_K D}{v}} = f\left(\underset{\phi}{\frac{\rho v D}{\mu}}, \underset{\phi}{\frac{D}{D}}\right) \Rightarrow \frac{F_K D}{v} = f\left(\frac{\rho v D}{\mu}\right)$$

(۳)

۶-(۶-۳۴)

$$L \qquad ML^{-3} \rightarrow MD^{-3}$$

$$\varphi(F_D, D, v, \rho, \mu)$$

$$DT^{-1} \leftarrow LT^{-1}$$

$$\begin{cases} L \doteq D \\ T \doteq DV^{-1} \\ M \doteq \rho D^{3} \end{cases}$$

$$F_D = MLT^{-2} \doteq \rho D^{3} \times D \times (DV^{-1})^{-2} = \rho D^{2} V^{2} \rightarrow \frac{F_D}{\rho D^{2} V^{2}}$$

$$\mu = ML^{-1}T^{-1} = \rho D^{3} \times D^{-1} \times D^{-1} V^{1} = \rho V D \rightarrow \frac{\mu}{\rho V D}$$

$\Delta P_l = f(Q, D, \mu)$ $\Delta P_l \doteq FL^{-3}, Q \doteq L^3T^{-1}, D \doteq L$ ۶-۴۰-

$, \mu \doteq FL^{-2}T$ و

با روش ایپسن پیش می‌رویم
متغیرهای تکراری Q, D و μ
هستند

$$\begin{cases} F \doteq \mu Q D^{-1} \\ L \doteq D \end{cases} \Rightarrow \Delta P_l \doteq (\mu Q D^{-1})(D)^{-3}$$

$$\doteq \mu Q D^{-4} \Rightarrow \Pi_1 = \frac{\mu Q}{\Delta P_l D^4}$$

$$\Rightarrow \frac{\Delta P_l}{\frac{D^4}{\mu Q}} = C \Rightarrow \begin{cases} D = 2\times10^{-3}\,m \\ \mu = 4\times10^{-3}\,Pa.s \\ l = 0.3\,m \end{cases} \quad \Delta P_l = \frac{\Delta P}{l}$$

$$\Rightarrow \begin{cases} C_1 = 40.74 \\ C_2 = 40.82 \\ C_3 = 40.21 \\ C_4 = 40.51 \\ C_5 = 40.82 \end{cases} \Rightarrow \overline{C} = 40.653$$

$$\Rightarrow \Delta P_l = 40.6\frac{Q\mu}{D^4}$$

مقیاس : $1:10 \Rightarrow \frac{x_m}{l_p} = 0.1 = L_r$ ۶-۴۵-

فرود $Fr : \left(\frac{V}{\sqrt{gl}}\right)_p = \left(\frac{V}{\sqrt{gl}}\right)_m \Rightarrow \frac{V_m}{V_p} = \sqrt{\frac{l_m}{l_p}} = L_r^{1/2}$

وبر $We : \left(\frac{\rho V^2 l}{\sigma}\right)_p = \left(\frac{\rho V^2 l}{\sigma}\right)_m \Rightarrow \frac{\sigma_m}{\sigma_p} = \left(\frac{V_m}{V_p}\right)^2 \cdot \frac{l_m}{l_p} = L_r^2 = \boxed{\frac{1}{100}}$

۶ـ ۴۹ـ

$P = f(\underset{\checkmark}{\rho}, \underset{\checkmark}{\omega}, \underset{\checkmark}{D}, h, Q)$, $P \doteq FLT^{-1}$ $\rho \doteq FL^{-4}T^{2}$

$\omega \doteq T^{-1}$, $D \doteq h \doteq L$, $Q \doteq L^{3}T^{-1}$

$$\left|\begin{array}{l} F \doteq \rho D^4\omega^2 \\ L \doteq D \\ T \doteq \omega^{-1} \end{array}\right. \Rightarrow \left|\begin{array}{l} P \doteq (\rho D^4\omega^2).(D)(\omega) = \rho D^5\omega^3 \\ h \doteq D \\ Q \doteq (D)^3.(\omega) = D^3\omega \end{array}\right.$$

$$\Pi_1 = \frac{\rho D^5\omega^3}{P}, \quad \Pi_2 = \frac{D}{h}, \quad \Pi_3 = \frac{D^3\omega}{Q} \Rightarrow \frac{P}{\rho D^5\omega^3} = \left(\frac{h}{D}, \frac{Q}{D^3\omega}\right)$$

$$\left(\frac{\rho D^5\omega^3}{P}\right)_m = \left(\frac{\rho D^5\omega^3}{P}\right)_p \Rightarrow \frac{\rho\times(0.8)^5\times 900^3}{5\times10^3} = \frac{\rho\times D_p^5\times 900^3}{50\times10^3} \Rightarrow D_p = 2.06\ m$$

$$L_r = \frac{D_m}{D_p} = \frac{0.8}{2.06} = 0.388, \left(\frac{D^3\omega}{Q}\right)_m = \left(\frac{D^3\omega}{Q}\right)_p$$

$$\Rightarrow \frac{0.8^3\times 900}{5} = \frac{(2.06)^3\times 900}{Q_p} \Rightarrow Q_p = 37.9\ L/s$$

$t = f(\underset{\checkmark}{d}, D, \underset{\checkmark}{\rho}, \mu, h, \underset{\checkmark}{g})$, $t \doteq T$, $d \doteq D \doteq h \doteq L$, $\rho \doteq FL^{-4}T^{2}$ ـ ۵۳ ـ ۶

$\mu \doteq FL^{-2}T$, $g \doteq LT^{-2}$ $\quad k = n - j = 7 - 3 = ④$

بدلیل اینکه از 3 بیشتر

شد از روش ساده‌تر (گام به گام) می‌رویم

$$1.\ \overset{T}{t} = f\left(\overset{\phi}{\frac{d}{d}}, \overset{\phi}{\frac{D}{d}}, \overset{FT}{\mu d^2}, \overset{\phi}{\frac{h}{d}}, \overset{T^{-2}}{\frac{g}{d}}\right)$$

۱ـ d برای حذف L

$$2.\ \overset{T}{t} = f\left(\overset{T^{-1}}{\frac{\mu}{\rho d^2}}, \overset{T^{-2}}{\frac{g}{d}}\right)$$

2ـ ρ برای حذف F

3ـ $\sqrt{\frac{g}{d}}$ برای حذف T

$$3.\ \overset{\phi}{t\sqrt{\frac{g}{d}}} = f\left(\overset{\phi}{\frac{\mu}{\rho d^2}\times\sqrt{\frac{d}{g}}}, \overset{\phi}{\frac{g}{d}\cancel{\times}\frac{d}{g}}\right) \Rightarrow \Pi_1 = \frac{\mu}{\rho\sqrt{\frac{d^4 g}{d}}} = \frac{\nu}{\sqrt{d^3 g}}$$

$$\Rightarrow \left(\frac{\nu}{\sqrt{d^3 g}}\right)_m = \left(\frac{\nu}{\sqrt{d^3 g}}\right)_p \Rightarrow \frac{\nu_m}{\nu_p} = \sqrt{\left(\frac{d_m}{d_p}\right)^3} = L_r^{3/2}$$

$$\Rightarrow \nu_m = (4.75\times 10^{-6})\times\left(\frac{1}{4}\right)^{3/2} = 5.94\times 10^{-7}\ m^2/s$$

$$\Pi_2 = t\sqrt{\frac{g}{d}} \Rightarrow \frac{t_m}{t_p} = \sqrt{\frac{d_p}{d_m}} = L_r^{-1/2}$$

با توجه به جدول این
لزجت سینماتیکی مربوط
به آب با دمای ۴۷°
است

$$\Rightarrow t_p = 4.53\times\left(\frac{1}{4}\right)^{-1/2} = 9.06\ min$$

سوال 6-58:

$$v_m = \frac{v_p}{l_r} = \frac{10}{1.10} \longrightarrow v_m = 100$$

$$F_{Dm} = \left(\frac{\rho_m}{\rho_p}\right)\left(\frac{v_m}{v_p}\right)^2\left(\frac{l_m}{l_p}\right)^2 \qquad F_{Dp} = \left(\frac{\rho_m}{\rho_p}\right)\left(\frac{1}{l_r}\right)^2(l_r)^2$$

$$F_{Dp} = (1)(1540) \longrightarrow \boxed{F_{Dm} = 1540}$$

سوال 6-63:

$$\frac{v_m}{v_p} = \left(\frac{\mu_m}{\mu_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{l_p}{l_m}\right) \qquad 1 = 1\left(\frac{\rho_p}{\rho_m}\right)(10)$$

$$\frac{p_m}{p_p} = 10 \qquad p_m = 10(101.33) \longrightarrow \boxed{p_m = 1013.3}$$

سوال 6-70:

$$\frac{v_m}{v_p} = \sqrt{\frac{l_m}{l_p}} = \frac{t_m}{t_p}\times\frac{x_m}{x_p} = \frac{v_m}{v_p} \qquad F_{rp} = \frac{v_p}{\sqrt{g_p l_p}} = F_{rm}$$

$$\frac{t_m}{24} = \sqrt{\frac{1}{500}} \longrightarrow t_m = 0.0..2\times 24 = 1.07h$$