تمرینات سری سوم مکانیک سیالات :

3-1)

میدان سرعت $\begin{cases} u = 6xt + y^2 z + 15 \\ v = 3xy^2 + t^2 + y \\ w = 2 + 3ty \end{cases}$ $\begin{cases} A(4,2,3)\ m \\ t = 1s \end{cases}$

Solution :

$$\begin{cases} u = 6(4m)(1s) + (2m)^2(3m) + 15 = 51\ m/s \\ v = 3(4)(2)^2 + (1)^2 + (2) = 51\ m/s \\ w = 2 + 3(1)(2) = 8\ m/s \end{cases}$$

$$\Rightarrow \boxed{\vec{V} = 51\,\hat{i} + 51\,\hat{j} + 8\,\hat{k}}$$

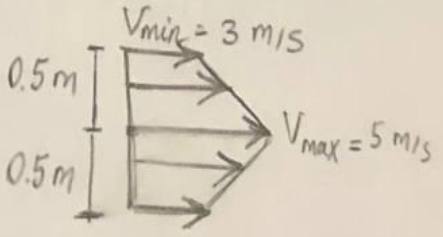
$$\Rightarrow |\vec{V}| = \sqrt{(51)^2 + (51)^2 + (8)^2} = \boxed{72.57\ m/s}$$

3-6) $Q = 0.35\ m^3/s$, $D = 500\ mm \Rightarrow \bar{V} = ?$

Solution : $\bar{V} = \dfrac{Q}{A} = \dfrac{0.35\ m^3/s}{\frac{\pi (0.5m)^2}{4}} = \boxed{1.78\ m/s}$

3-12) $D = 1\ m$, $V_{max} = 5\ m/s$, $V_{min} = 3\ m/s \Rightarrow Q = ?$

Solution : $Q = \int_A (\vec{V}\cdot\vec{n})\, dA$



$$\vec{V}\cdot\vec{n} = u = 5 - \frac{2r}{R} = 5 - \frac{2r}{(0.5m)} = 5 - 4r$$

$$Q = \int_A u\, dA = \int_A (5-4r)(2\pi r\, dr) = 2\pi \int_{r=0}^{r=R} (5r - 4r^2)\, dr = 2\pi \left[\frac{5}{2}r^2 - \frac{4}{3}r^3\right]_{r=0}^{r=R}$$

$$Q = 2\pi \left[(2.5)(0.5m)^2 - \frac{4}{3}(0.5)^3\right] = \boxed{2.88\ m^3/s}$$

3-18) $\begin{cases} T_1 = 300\ K \\ P_1 = 690\ kPa \\ \bar{V}_1 = ? \end{cases}$ $\begin{cases} T_2 = 252\ K \\ P_2 = 127\ kPa \\ \bar{V}_2 = 320\ m/s \end{cases}$ $D = 300\ mm$

رابطه پیوستگی :-

Solution: $\frac{\partial}{\partial t}\int_{CV} \rho\, d\forall + \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA = 0 \Rightarrow \frac{d}{dt}(M) - \rho_1 \bar{V}_1 A_1 + \rho_2 \bar{V}_2 A_2 = 0$ (1)

$$\rho_1 = \frac{P_1}{RT_1} = \frac{690\times10^3\ Pa}{(270\ J/kg\cdot K)(300\ K)} = 8.52\ kg/m^3$$

$$\rho_2 = \frac{P_2}{RT_2} = \frac{127\times10^3\ Pa}{(270\ J/kg\cdot K)(252\ K)} = 1.87\ kg/m^3$$

(1) $\Rightarrow \rho_1 \bar{V}_1 A_1 = \rho_2 \bar{V}_2 A_2 \Rightarrow \bar{V}_1 = \frac{\rho_2}{\rho_1}\bar{V}_2 = \left(\frac{1.87}{8.52}\right)(320\ m/s)$

$\Rightarrow \boxed{\bar{V}_1 = 70.1\ m/s}$

3-22) $V_z = 0.4\left[1-\left(\frac{r}{R}\right)^2\right]\ m/s$, $\rho = 1000\ kg/m^3$

$D = 6\ cm$, $\dot{m} = ?\ kg/hr$

Solution:

$$\dot{m} = \rho Q = \rho \int V_z\, dA = \rho \int_0^R 0.4\left[1-\left(\frac{r}{R}\right)^2\right](2\pi r\, dr)$$

$$\dot{m} = (0.8\pi)(\rho)\int_0^R \left[r - \frac{r^3}{R^2}\right] dr = (0.8\rho\pi)\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]\Bigg|_{r=0}^{r=R}$$

$$\dot{m} = (0.8\rho\pi)\left(\frac{R^2}{4}\right) = (0.8)(1000\ kg/m^3)(\pi)\frac{(0.03m)^2}{4} = 0.57\ kg/s$$

$$\Rightarrow \dot{m} = 0.57\ kg/s \times \frac{3600\ s}{1\ hr} = \boxed{2035.75\ kg/hr}$$

3-28) $V_1 = 8\ m/s$, $D = 0.3\ m$, $\ell = 2m$

$U = 0.4\, U_2 \left(\frac{x}{\ell}\right)$ — رابطه ۲ صحیح است.

Solution : $\sum_{in} \dot{m} = \sum_{out} \dot{m} \Rightarrow \dot{m}_1 = \dot{m}_2 + \dot{m}_3$ (1)

$\dot{m}_1 = \rho \bar{V}_1 A_1 = (1000\ kg/m^3)(8\ m/s)\left(\frac{\pi}{4}(0.3m)^2\right) = 565.5\ kg/s$

$\dot{m}_2 = \rho \bar{V}_2 A_2 = (1000\ kg/m^3)(U_2)\left[\frac{\pi}{4}(0.3)^2\right] = 70.69\, U_2\ kg/s$

$\dot{m}_3 = \int_{A_3} \rho(\vec{V}\cdot\vec{n})\, dA = \rho \int_0^2 (0.4U_2)\left(\frac{x}{\ell}\right)(2\pi R\, dx)$

$= \frac{0.8\pi\rho U_2 R}{\ell} \int_0^{\ell} x\, dx = \frac{0.4\pi\rho U_2 R}{\ell} x^2 \Big|_{x=0}^{x=\ell}$

$= \frac{(0.4\pi)(1000\ kg/m^3)\, U_2\, (0.15m)}{(2m)} (2m)^2 = 377\, U_2\ kg/s$

(1) $\Rightarrow 377\, U_2 + 70.69\, U_2 = 565.5 \Rightarrow U_2 = \frac{565.5}{447.7} = 1.263\ m/s$

$\Rightarrow \dot{m}_3 = 377\,(1.263\ m/s) = \boxed{476.2\ kg/s}$

3-33) Solution : $\frac{\partial}{\partial t}(A_A h_A) + \sum_{out} Q - \sum_{in} Q = 0$ — نرخ آبدهی ورودی از مخزن A

$\Rightarrow (1.5m)(1.5m)\frac{dh_A}{dt} + (0.15\ m^3/s) - (0.18\ m^3/s) = 0 \Rightarrow \frac{dh_A}{dt} = 13.3\ mm/s$

مخزن B : $\frac{\partial}{\partial t}(A_B h_B) + \sum_{out} Q - \sum_{in} Q = 0 \Rightarrow (2.5)(1.5)\left(\frac{-0.06\ m}{1\ s}\right) + Q_3 - Q_2 = 0$

$\Rightarrow Q_3 = (0.15\ m^3/s) + (0.225\ m^3/s) = \boxed{0.375\ m^3/s}$

3-38) $\forall = 10\ m^3$, $\dot{m} = 0.5\rho_0/\rho$, $\rho_0 = 2\ kg/m^3$

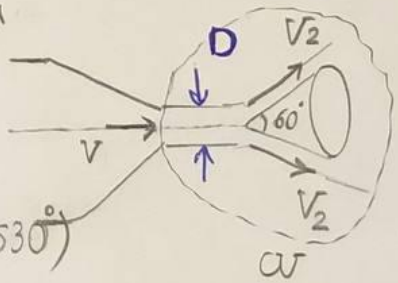
Solution: $\frac{\partial}{\partial t}\int_{CV}\rho\, d\forall - \dot{m} = 0 \Rightarrow \frac{d}{dt}(\rho\forall) - \dot{m} = 0$

$\Rightarrow \frac{d\rho}{dt} = -\frac{\dot{m}}{\forall} \Rightarrow \frac{d\rho}{dt} = -\frac{0.5\rho_0/\rho}{\forall}$

$\int_{\rho_0}^{\rho_f}\rho\, d\rho = \int_0^{\Delta t}\frac{0.5\rho_0}{\forall}\, dt \Rightarrow 0.5(\rho_f^2 - \rho_0^2) = \frac{0.5\rho_0}{\forall}\Delta t$

$\Rightarrow \Delta t = \forall\rho_0\left[\left(\frac{\rho_f}{\rho_0}\right)^2 - 1\right] = (10m^3)(2kg/m^3)(2^2-1) = 60\ s$

3-45) $\sum_{in} Q = \sum_{out} Q \Rightarrow 2Q_2 = Q_1$



Solution: $\sum_{CV} F_x = \sum_{CS}\rho Q V_x$

$\sum_{CV} F_x = (-\rho Q_1)(V) + 2\left(\rho\frac{Q_1}{2}\right)(V\cos 30^\circ)$

$\sum_{CV} F_x = \left(\frac{\sqrt{3}}{2} - 1\right)(\rho Q_1 V) = \left(\frac{\sqrt{3}}{2} - 1\right)\rho\left(\frac{\pi D}{4}\right)^2 V^2 = -0.105\,\rho V^2 D^2$

3-51) $D = 20cm$, $V =$ m/s , $g = 10\ m/s^2$, $\rho = 1000\ kg/m^3$

Solution: $\sum_{CV} F_x = \sum_{CS}\rho Q V_x = \rho \cdot A_j \cdot V_j^2 = (1000)\left[\frac{\pi(0.2)^2}{4}\right](20)^2 = 4\pi\ kN$

شرط تعادل لنگر حول نقطه A: $\Sigma M_A = 0 \Rightarrow F_j \times \frac{h}{2} = F_B \times h$

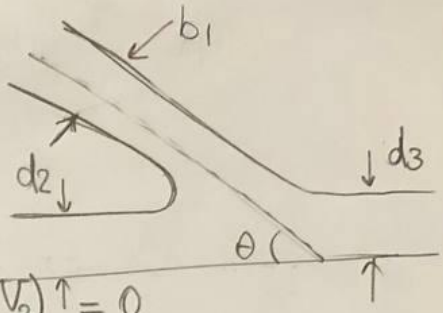
AB

$\Rightarrow F_B = \frac{F_j}{2} = 2\pi = 6.28\ kN$

گزینه ۱ صحیح است.

3-56)

Solution : $\sum_{CS} \rho Q V_x = \sum_{CV} F_x = 0$



$\Rightarrow (-\rho Q_1)(+V_1\cos\theta)+(\rho Q_2)(-V_2)+(\rho Q_2)(V_2)\uparrow = 0$

$\Rightarrow -V^2 A_1\cos\theta - V^2 A_2 + V^2 A_3 = 0 \Rightarrow -b_1\cos\theta - d_2 + d_3 = 0$

$\Rightarrow d_2 = d_3 - b_1\cos\theta \quad (1)$

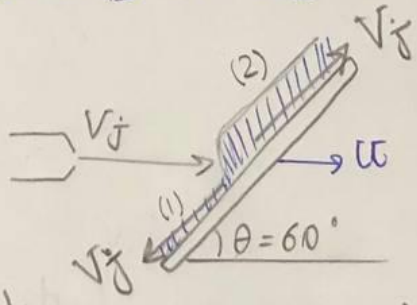
زیبنی آب صحیح است.

رابطه پیوستگی : $Q_1 = Q_2 + Q_3 \Rightarrow b_1 = d_2 + d_3$

$\Rightarrow b_1 = (d_3 - b_1\cos\theta) + d_3 \Rightarrow \boxed{d_3 = \frac{b_1}{2}(1+\cos\theta)}$

3-63) $A_j = 700\ mm^2$, $V_j = 20\ m/s$, $u = 5\ m/s \Rightarrow \Sigma F_x = ?$

Solution : $\int \rho V_x \vec{V}.\vec{n}\,dA = \Sigma F_x$



$\Rightarrow R_x = \Sigma \rho Q u$

$\Rightarrow -\rho Q_1(V_j - u) + \rho Q_2(V_j\cos 60 - u) + \rho Q_3(-V_j\cos 60 - u) = R_x$

$Q_2 = \frac{2}{3}Q_1$, $Q_3 = \frac{1}{3}Q_1$, $Q_1 = A_j V_j = 1.4\times10^{-2}\ m^3/s$

$\Rightarrow R_x = 10^3 \times Q_1\left[-15 + (\frac{2}{3})(5) - (\frac{1}{3})(+15)\right] = \boxed{-233.3\ N}$

3-69) $a = \frac{\rho Q}{M}\left[(V_j - V_c)\left(1 - \frac{V_c}{V_j}\right)\right]$

Solution: $\int_{CS} V_{rx}\,\rho(\vec{V}\cdot\vec{n})\,dA = \sum_{CV} F_x$

$\Rightarrow (V_j - V_c)\rho(V_j - V_c)A_j = Ma$

$\Rightarrow Ma = \rho A_j V_j (V_j - V_c)\left(1 - \frac{V_c}{V_j}\right)$

$\Rightarrow \boxed{a = \frac{\rho Q}{M}\left[(V_j - V_c)\left(1 - \frac{V_c}{V_j}\right)\right]}$