حل تمرینات سری دوم مکانیک سیالات :

2-1)

حل : گزینه‌ی ۴ ، اگر تنش برشی صفر باشد ، فشار در همه‌ی جهات برابر خواهد بود.

2-10)

$$\text{حل: } E_V = \frac{dP}{dP/\rho} \Rightarrow \frac{d\rho}{\rho} = \frac{dP}{E_V} \cong \frac{\Delta P}{E_V} = \frac{\gamma \Delta h}{E_V}$$

$$\Rightarrow \frac{d\rho}{(1025\ kg/m^3)} = \frac{[(1025\ kg/m^3)(9.81\ m/s^2)](1000\ m)}{(2.35\times10^9\ N/m^2)}$$

$$\Rightarrow \Delta\rho = 4.387\ kg/m^3 \Rightarrow \rho_{1km} = (1025\ kg/m^3) + (4.387\ kg/m^3)$$

$$\Rightarrow \boxed{\rho_{1km} = 1029.4\ kg/m^3}$$

2-15)

$$\text{حل: } P = \rho g h + P_{atm} = (850\ kg/m^3)(9.81\ m/s^2)(0.45\ m) + (98\times10^3\ Pa)$$

$$\Rightarrow \boxed{P_{\text{مطلق}} = 101.75\ kPa}$$

2-23)

حل : فشارسنجی در نقاط A و B بصورت زیر بدست می آید :

$$P_A + (0.65m)(\gamma w) - (0.65m)(\gamma w) - (0.45m)(\gamma w) = 0$$

$$\Rightarrow \boxed{P_A = 4414.5\ Pa}$$

$$P_B + (0.5m)(\gamma w) - (0.5m)(\gamma w) - (0.6m)(\gamma w) = 0$$

$$\Rightarrow \boxed{P_B = 5886\ Pa}$$

2-27)

$$\text{در: } P_{1-1} = (0.13m)\gamma_w = (0.18m)\gamma_f$$

$$\Rightarrow \gamma_f = \frac{(0.13m)\gamma_w}{(0.18m)} = \boxed{7085\ N/m^3}$$

2-34)

$$\text{در: } P_A + (0.6m)(\rho_{\text{آب شیرین}})g - (0.1m)(\rho_{\text{جیوه}})g + (0.4m)(\rho_{\text{آب دریا}})g = P_B$$

$$\Rightarrow P_A - P_B = -(0.6m)(1000\ kg/m^3)(9.81) + (0.1m)(13.6 \times 1000\ kg/m^3)(9.81)$$

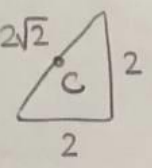
$$-(0.4m)(1035\ kg/m^3)(9.81) \Rightarrow P_A - P_B = -3394.26\ Pa$$

$$\boxed{P_A - P_B = -3.39\ kPa}$$

2-38) گزینه ۳

$$\text{در: } F_R = \gamma_w h_c A = \gamma_w (2m)\left[\pi (1m)^2\right] = \boxed{2\pi\gamma_w}$$

2-46)



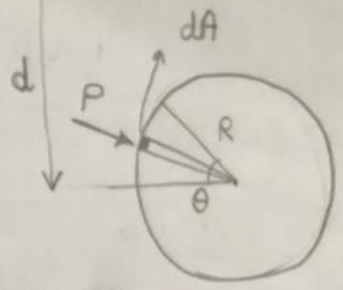
$$F = (P_o + \gamma h_c)\, A$$

$$F = \left[100{,}000\ Pa + (9810\ N/m^3)(9m)\right]\left((2\sqrt{2} \times 5)\ m^2\right)$$

$$\boxed{F = 2663\ kN}$$

2-52) حل: گزینه‌ی ۳ صحیح است، زیرا اگر دریچه در مرکز باشد، نقاط اعمال نیروی فشاری و دریچه هم منطبق می‌شوند و بازوی نیرو صفر می‌شود. پس افزایش یا کاهش H، تأثیری بر T_A ندارد. امّا با افزایش H، نیروی فشاری اعمال شده به نقطه‌ی B دریچه افزایش پیدا می‌کند که این امر باعث افزایش T_B می‌گردد.

2-58) h - فاصله‌ی آن تا سطح آب



حل: $dA = (2\pi R\cos\theta)(Rd\theta)$

$dA = 2\pi R^2 \cos\theta\, d\theta$

$h = d - R\sin\theta \Rightarrow F_z = \int_A -\gamma h \sin\theta\, dA$

$$F_z = \int_{-\frac{\pi}{2}}^{\pi/2} -\gamma(d - R\sin\theta)\sin\theta\,(2\pi R^2\cos\theta\, d\theta)$$

$$F_z = -2\pi R^2\gamma \int_{-\pi/2}^{\pi/2} \cos\theta\sin\theta\, d\theta + 2\pi R^3\gamma \int_{-\pi/2}^{\pi/2} \cos\theta\sin^2\theta\, d\theta$$

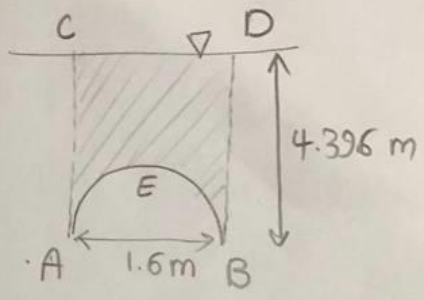
$$F_z = -2\pi R^2\gamma\, d\left[\frac{1}{2}\sin^2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + 2\pi R^3\gamma\left[\frac{1}{3}\sin^3\theta\right]_{-\frac{\pi}{2}}^{\pi/2}$$

$$F_z = -2\pi R^2\gamma\, d(1-1) + 2\pi R^3\gamma\left[\frac{1}{3}(1-(-1))\right]$$

$$\boxed{F_z = \frac{4}{3}\pi R^3\gamma}$$

2-66)

$$h = \frac{p}{\gamma_{oil}} = \frac{(60000\ N/m^2)}{(0.9 \times 9810\ N/m^3)} = 6.8\ m$$



$$F_V = \gamma(\forall_{AEBDC}) = SG\gamma_w(\forall_{ABCD} - \forall_{AEB})$$

$$F_V = \left[(0.9)(9810\ N/m^3)\right]\left[(1.6\,m)(3m)(4.396\,m) - \left[\frac{\pi}{2}\frac{(1.6m)^2}{4}(3m)\right]\right]$$

$$F_V = 159671\ N = 159.7\ kN$$

71-2) حل: گزینه‌ی ۴ درست است.

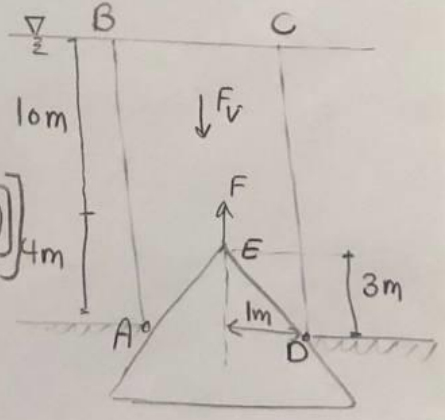
$$N_B = W - F_V = W - \gamma \forall_{BDC} = W - \gamma\left[\left(\frac{1}{2}\pi R^2\right)(\ell)\right]$$

$$N_B = \left(200{,}000\ N - \left((1000\ kg/m^3)(10\ m/s^2)\right)\left[\frac{1}{2}\pi\left(\frac{3}{2}m\right)^2(4\,m)\right]\right.$$

$$\Rightarrow N_B = 58.6\ kN$$

78-2) $$h = \frac{p}{\gamma} = \frac{(100{,}000\ N/m^2)}{(10{,}000\ N/m^3)} = 10\ m$$

حل: گزینه‌ی ۴ صحیح است.



$$F = F_V = \gamma(\forall_{AGBDCA}) = \gamma(\forall_{ABDC} - \forall_{AGB})$$

$$F_V = (10{,}000\ N/m^3)\left[\left[\pi(1m)^2(14m)\right] - \left[\frac{1}{3}\pi(1m)^2(3m)\right]\right]$$

$$F = 130{,}000\,\pi\ N$$

حل: گزینه ۲ صحیح است.

2-84)

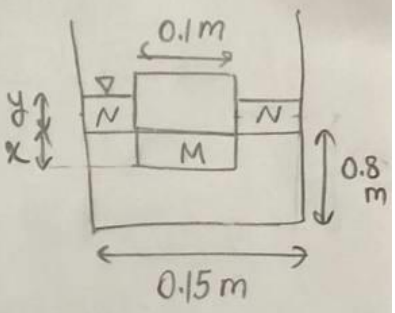
$$F_2 = \gamma_w A h_c = \gamma\left[(\ell)(1m)\right](2m) = 2\ell\gamma_w$$

$$F_3 = \gamma A h_c = \gamma\left[(2m)(1m)\right]\left(\frac{2}{2}m\right) = 2\gamma_w$$

$$\Sigma M_O = 0 \Rightarrow F_2\left(\frac{\ell}{2}\right) - F_3\left(\frac{2r}{3}\right) = 0 \Rightarrow (2\ell\gamma_w)\left(\frac{\ell}{2}\right) - (2\gamma_w)\left(\frac{4}{3}m\right) = 0$$

$$\Rightarrow \boxed{\ell = 1.63\ m}$$

2-92) $V_m = V_n$:طرح



$$\Rightarrow (0.1m)(0.1m)x = \left[2(0.15m) + 2(0.1m)\right](0.05)y$$

$$\Rightarrow x = 2.5y$$

$$F_B = W \Rightarrow \gamma_{liq}\left[(0.1m)(0.1m)\right](x+y) = (5\ N)$$

$$\Rightarrow \left[(0.8)(9810\ N/m^3)\right]\left[(0.1m)(0.1m)\right](2.5y + y) = 5\ N$$

$$y = 0.018\ m = 18\ mm \Rightarrow \text{ارتفاع بالا آمدگی} = y + 0.08 = \boxed{98\ mm}$$

2-97) طرح: $D = 1.5\ m$, $T = 5.3\ kN$ $\Rightarrow W = ?$

$$T = W - F_B \Rightarrow W = T + F_B = (5.3 \times 10^3\ N) + \left(\frac{\pi}{6}\gamma_w D^3\right)$$

$$\Rightarrow W = (5300\ N) + \frac{\pi}{6}(9810\ N/m^3)\left(1.5_m\right)^3 = 22{,}635.7\ N$$

$$\Rightarrow \boxed{W = 22.6\ kN}$$

2-102) داده: $\tan\theta = \frac{12}{20} \Rightarrow \theta = 30.96^\circ$, $d = 2y\tan\theta$

$$F_B = W \Rightarrow \gamma_w\left(\frac{1}{3}\frac{\pi d^2}{4}y\right) = (SG\,\gamma_w)\left(\frac{1}{3}\frac{\pi D^2}{4}H\right)$$

$$\Rightarrow d^2y = SGD^2H \Rightarrow (2y\tan\theta)^2 y = SG(2H\tan\theta)^2 H$$

$$\Rightarrow y^3 = SGH^3 \Rightarrow y = SG^{1/3}H = (0.8)^{1/3}(0.2m) = 0.186\ m$$

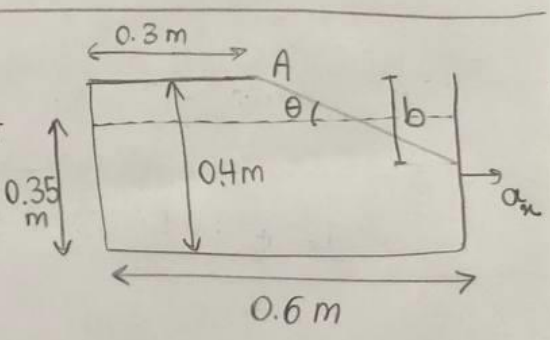
$$\overline{BG} = \overline{OG} - \overline{OB} = \left(\frac{3}{4}H\right) - \left(\frac{3}{4}y\right) = \frac{3}{4}\left[(0.2m) - (0.186m)\right] = 0.011\ m$$

$$\overline{GM} = \overline{BM} - \overline{BG} = \frac{I}{\forall} - \overline{BG} = \frac{(\pi d^4/64)}{1/3\,(\pi d^2/4)y} - \overline{BG} = \frac{3}{16}\frac{(2y\tan\theta)^2}{y} - \overline{BG}$$

$$\overline{GM} = \frac{3}{4}y\tan^2\theta - \overline{BG} = \frac{3}{4}(0.186m)\tan^2(30.96^\circ) - (0.011\ m)$$

$$\Rightarrow \overline{GM} = 0.039\ m > 0 \Rightarrow$$ جسم پایداری ماند.

5-35) ω - عرض مخزن



$$(0.35m)(0.6m)\omega = \left[(0.6m)(0.4m) - \frac{(0.3m)b}{2}\right]\omega$$

$$\Rightarrow 0.21 = 0.24 - 0.15b \Rightarrow b = 0.2m$$

$$\tan\theta = \frac{dz}{dx} = \frac{-a_x}{g} = -\frac{(0.2m)}{(0.3m)}$$

$$\Rightarrow a_x = \frac{2}{3}g = \frac{2}{3}(9.81\ m/s^2) \Rightarrow \boxed{a_x = 6.56\ m/s^2}$$

5-41)

حل: $\tan\theta = \frac{dz}{dy} = \frac{-a_y}{g} = -\frac{(2.5\ m/s^2)}{(9.81\ m/s^2)} \Rightarrow \tan\theta = -0.255$

مقدار پایین آمدن آب در سطح جلویی مخزن $= (4m) \times 0.255 = 1.02\ m \Rightarrow$ ارتفاع سطح جلویی $= 2 - 1.02 = 0.98\ m$

$\Rightarrow$ ارتفاع سطح عقبی $= 3.02\ m$

$$\begin{cases} F_{front} = \gamma h_c A = (9810\ N/m^3)\ \frac{(0.98m)}{2}\ [(0.98m)(2m)] = 9434\ N \\ F_{back} = \gamma h_c A = (9810\ N/m^3)\left(\frac{3.02m}{2}\right)[(3.02m)(2m)] = 89434\ N \end{cases}$$

$\Rightarrow \boxed{\Delta F = 80{,}000\ N}$

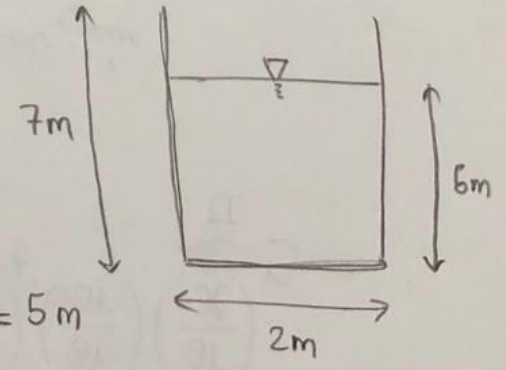
$F = ma = \rho \forall a = (1000\ kg/m^3)[(8m)(2m)(2m)](2.5\ m/s^2)$

$\Rightarrow \boxed{F = 80{,}000\ N}$

5-44)

$$\begin{cases} d = 2m & g = 10\ m/s^2 \\ \omega = 10\ rad/s \end{cases}$$

حل: گزینه‌ی ۳ صحیح است.



$$2h = z = \frac{\omega^2 r^2}{2g} = \frac{(10\ rad/s)^2 (1m)^2}{2(10)} = 5\ m$$

$\Rightarrow h = 2.5\ m \Rightarrow$ به اندازه‌ی ارتفاع $0.5\ m$ آب از مخزن خارج می‌شود.

$\Rightarrow \forall_{\text{خارج شده}} = (0.5m)\left(\frac{\pi (2m)^2}{4}\right) = \boxed{1.57\ m^3}$

5-49)

$$P_B - P_A = \frac{\rho\omega^2 r_B^2}{2} - \gamma z_B + C_5 - \left(\frac{\rho\omega^2 r_A^2}{2} - \gamma z_A + C_5 \right)$$

$$P_B - P_A = \frac{\rho\omega^2}{2}\left(r_B^2 - r_A^2\right) = \frac{\left[(0.9)(9810\ kg/m^3)\right](40\ rad/s)^2}{2}\left[(0.2m)^2 - 0\right]$$

$$P_B - P_A = 141{,}264\ Pa = 141.3\ kPa$$