حل مسائل سری اوّل مکانیک سیالات

1-4) $\begin{cases} u = 4zt \\ v = 3x \\ w = 0 \end{cases}$

حل: جریان دوبعدی، غیرماندگار و غیرتکنواخت

1-6) الف) $(14.2\ \text{km})\left(1000\ \frac{\text{m}}{\text{km}}\right)\left(3.281\ \frac{\text{ft}}{\text{m}}\right) = 46590.2\ \text{ft}$

ب) $\left(8.14\ \frac{\text{N}}{\text{m}^3}\right)\left(6.366\times10^{-3}\ \frac{\text{lb/ft}^3}{\text{N/m}^3}\right) = 5.18\times10^{-2}\ \text{lb/ft}^3$

ج) $(160\ \text{acre})\left(4046.873\ \frac{\text{m}^2}{\text{acre}}\right) = 6.47\times10^{5}\ \text{m}^2$

د) $(742\ \text{Btm})\left(1.055\times10^{3}\ \frac{\text{J}}{\text{Btm}}\right) = 7.83\times10^{5}\ \text{J}$

1-12)

حل: گزینه‌ی ۳. هر ۳ تا همگن هستند.

$\Delta h \doteq D \doteq L \quad ; \quad \gamma \doteq FL^{-3} \quad ; \quad \sigma \doteq FL^{-1}$

1) $\Delta h = D\,\xi\left(\frac{\gamma D^2}{\sigma}\right) : (L) \doteq (L)\left[\frac{(FL^{-3})(L)^2}{(FL^{-1})}\right] \doteq (L)$ ✓

2) $\Delta h\sqrt{\frac{\gamma}{\sigma}} = \sum\left(\frac{\sigma}{\gamma D^2}\right) : (L)\left(\frac{FL^{-3}}{FL^{-1}}\right)^{1/2} \doteq \left[\frac{(FL^{-1})}{(FL^{-3})(L)^2}\right]$

$\Rightarrow (L)(L^{-1}) \doteq F^0L^0T^0$ ✓

3) $\frac{\Delta h}{D} = f\left(\frac{\sigma}{\gamma D^2}\right) : \frac{(L)}{(L)} \doteq \left[\frac{(FL^{-1})}{(FL^{-3})(L)^2}\right]$

$\Rightarrow F^0L^0T^0 \doteq F^0L^0T^0$ ✓ پس هر سه رابطه از لحاظ ابعادی همگن هستند

1-18) $\frac{\partial^2 x}{\partial t^2} + A\frac{\partial x}{\partial t} + Bx = 0$

do: $x \doteq L \;;\; t \doteq T^1$

$$\frac{L}{T^2} + A\frac{L}{T} + B(L) = 0$$

$$A \doteq \frac{LT^{-2}}{LT^{-1}} \doteq \boxed{T^{-1}} \;;\; B \doteq \frac{LT^{-2}}{L} \doteq \boxed{T^{-2}}$$

1-23) do: $\frac{P_1 \forall_1}{T_1} = \frac{P_2 \forall_2}{T_2} = \frac{P_2 \forall}{T_2}$

$$\Rightarrow P_2 = P_1\frac{T_2}{T_1} = \left[(210+101)\,kPa\right]\frac{(50+273)\,K}{(25+273)\,K} = \boxed{336\ kPa} = P_2$$

$$P = \rho RT = \frac{m}{\forall}RT \Rightarrow m = \frac{P\forall}{RT}$$

$$\Rightarrow \begin{cases} m_1 = \dfrac{P_1\forall}{RT_1} = \dfrac{(310\,kPa)(0.025\,m^3)}{(0.287\,kP_a\cdot m^3/kg\cdot K)(323\,K)} = 0.0906\ kg \\ m_2 = \dfrac{P_2\forall}{RT_2} = 0.0836\ kg \end{cases}$$

$$\Rightarrow \Delta m = m_1 - m_2 = \boxed{0.007\ kg = 7\,gr}$$

1-29) do: $F_\tau = \int_A \tau\, dA = 2\int_0^\ell \left(\mu\frac{du}{dy}\right)(b\,dx) = 2b\mu\int_0^\ell \frac{d}{dy}\left(\frac{Uy}{\delta}\right)dx$

$$F_\tau = 2bU\mu\int_0^\ell \frac{1}{\delta}dx = 2bU\mu\int_0^\ell \frac{1}{3.5\sqrt{\nu x/U}}dx = \frac{2}{3.5}bU\mu\int_0^\ell \left(\frac{U}{\nu x}\right)^{1/2}dx$$

$$F_\tau = \frac{2}{3.5}\frac{bU^2\mu}{\sqrt{\nu}}\int_0^\ell x^{-1/2}dx = \frac{2}{3.5}\sqrt{\rho U}\,bU^{3/2}\int_0^\ell x^{-1/2}dx = \frac{2}{3.5}\sqrt{\rho U}\,bU^{3/2}(2\ell^{1/2})$$

$$\Rightarrow \boxed{F_\tau = 1.143bU\sqrt{\rho\mu\ell U}}$$

1-33) do: $\tau = \mu \frac{du}{dy} = \mu \frac{d}{dy}\left[-\frac{1}{2}\frac{\gamma}{\mu}\frac{dP}{dx}(Hy - y^2) + U_t \frac{y}{H}\right]$

$$\tau = -\frac{\gamma}{2}\frac{dP}{dx}(H-2y) + \frac{\mu U_t}{H}$$

$$\begin{cases} \tau_u = -\frac{\gamma}{2}\frac{dP}{dx}(H-2H) + \frac{\mu U_t}{H} \\ \tau_d = -\frac{\gamma}{2}\frac{dP}{dx}(H-0) + \frac{\mu U_t}{H} \end{cases} \Rightarrow \begin{cases} \tau_u = \frac{\gamma}{2}\frac{dP}{dx}H + \frac{\mu U_t}{H} \\ \tau_d = -\frac{\gamma}{2}\frac{dP}{dx}H + \frac{\mu U_t}{H} \end{cases}$$

1-38) do: $F = (\tau_1 + \tau_2)A = A\left[\mu_1 \frac{du_1}{dy_1} + \mu_2 \frac{du_2}{dy_2}\right]$

$$F = A\left[(4\mu_2)\frac{U-0}{(L-x)} + \mu_2 \frac{U-0}{x}\right] = AU\mu_2\left[\frac{4}{L-x} + \frac{1}{x}\right]$$

$$\frac{dF}{dx} = \frac{d}{dx}\left[U\mu_2\left(\frac{4}{L-x} + \frac{1}{x}\right)\right] = 0$$

$$\Rightarrow \frac{4x + (L-x)}{x(L-x)} = 0 \Rightarrow x = \frac{1}{3}L$$

1-41)

$$T = \frac{2\pi\mu\omega L R_i^3}{(R_o - R_i)}$$

1-46)

$$\begin{cases} dT = r\,dF = r(\tau\,dA) \\ r = R\sin\theta \\ \tau = \mu\frac{du}{dr} = \mu\frac{V}{\Delta r} = \mu\frac{\omega R\sin\theta}{t} \\ dA = 2\pi r(ds) = 2\pi R\sin\theta(R\,d\theta) = 2\pi R^2\sin\theta\,d\theta \end{cases}$$

$$dT = (R\sin\theta)\left(\mu\frac{\omega R\sin\theta}{t}\right)(2\pi R^2\sin\theta\,d\theta) \Rightarrow T = 1.97\times10^{-3}\ N.m$$

1-52)

$$\text{حل: } E_V = \frac{dP}{d\rho/\rho} = \frac{\Delta P}{\Delta\rho/\rho} \Rightarrow (2.5\times10^{9}\ Pa) = \frac{10\times10^{6}\ Pa}{\Delta\rho/\rho}$$

$$\Rightarrow \boxed{\frac{\Delta\rho}{\rho} = 0.004}$$

1-55)

$$\text{حل: } P_2 = P_1\frac{\forall_1}{\forall_2} = P_1\frac{\forall_1}{(0.3\forall_1)} = \frac{1}{0.3}P_1 = \frac{1}{0.3}(172\ kPa) = \boxed{573.3\ kPa}$$

1-63)

حل: بله، چون فشار داخل = 2.34 kPa > 2 kPa

1-70)

$$\text{حل: } h = \frac{2\sigma\cos\phi}{\gamma R} = \frac{2(0.073\ N/m)\cos(15^\circ)}{(9789\ N/m^3)(5\times10^{-6}\ m)} = \boxed{5.8\ m}$$